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1. Introduction

Rational expectations are becoming increasingly popular as a means of generating unobservable explanatory variables in econometric models (Nelson [1975a], McCallum [1976a], Sargent [1976] and Weaver and Helmberger [1977]). The intuitive attractiveness of the rational expectations hypothesis advanced by Muth [1961] has been evident for some time. However, problems of implementing the hypothesis in applied contexts have limited its use. Specifically, for more than highly simplified constructs, methods of obtaining estimates of the unobservable explanatory variables implied by the systematic component of the model, and thus rational, but permitting consistent estimation of the structural parameters have not been available.

This difficulty with the application of the rational expectations hypothesis in econometric models was highlighted in a recent exchange by Nelson [1975b] and McCallum [1976b]. Moreover, building on the results by Nelson, McCallum was able to show that an instrumental variables approach using extrapolative prediction for the unobservable variables would produce consistent estimates for the structure of a model incorporating "partially" rational expectations. The expectations are termed partial because the extrapolative process for generating the instruments does not incorporate all of the restrictions on the unobservable variables which are implied by the structure.

The present paper extends these results by showing that the extrapolative process imposed by McCallum for obtaining consistent estimates of the structural parameters with a rational expectations hypothesis on the unobservable variables is unnecessary. Namely, more direct methods for producing the required unobservables and correspondingly consistent estimates of the structural parameters can be derived. These include the unobservable variables (Goldberger [1972], Zellner [1970]), instrumental variables (Brundy and Jorgenson [1971b, 1974]) and iterated instrumental variables (Lyttkens [1974]) estimation methods. Each of the methods provides an operational means of obtaining consistent estimates in models with rational expectations.

The model and process for incorporating the rational expectations hypothesis in estimating the structural parameters follow in Section 2. Basic consistency results for the corresponding parameter estimates are provided in Section 3. Various specializations of the fully rational expectations hypothesis are examined in Section 4. The objective of this section is to identify the circumstances under which expectations hypotheses common to applied work are in fact rational. The approach is illustrated with a simple supply and demand model in Section 5. Concluding observations are provided in Section 6.

2. Model and Rational Expectations Approach

The rational expectations hypothesis (Muth [1961]) puts structure on the unobservable expectations variables which are frequently incorporated in economic constructs by assuming that they are in fact the expected values of the corresponding endogenous variables as predicted by the model.[1] The rational expectations approach has been less widely applied than some of

the adaptive or extrapolative methods because the usually complex structures for generating the required unobservables have proved difficult to implement while retaining desirable statistical properties for the parameter estimates. The subsequent discussion is designed to show that the incorporation of the rational expectations hypothesis can be more easily handled than presently available results (Nelson [1975a] and McCallum [1976b]) would indicate.

The model to be used is of the standard linear form where y_t, x_t and y^*_{t+i} denote the t^{th} observations on G observable endogenous, K observable predetermined, and M unobservable expectations variables, respectively.[2] All of the variables are taken as expressed in deviation form. It is assumed that expectations are generated about n future periods, $i = 1, 2, \ldots, n$. That is, the "planning period" for the system is up to n periods in length.[3] The linear model relating these observable and unobservable variables and the structural disturbance vector u_t is, for the t^{th} observation,

$$(1) \qquad y_tB + x_t\Gamma + y^*_{t+1}\Delta_1 + y^*_{t+2}\Delta_2 + \ldots + y^*_{t+n}\Delta_n + u_t = 0$$

where B, Γ and Δ_i ($i = 1, 2, \ldots, n$) are appropriately dimensioned matrices of structural parameter. For all (T) observations, model (1) can be written

$$(1') \qquad YB + X\Gamma + Y^*_1\Delta_1 + Y^*_2\Delta_2 + \ldots + Y^*_n\Delta_n + u = 0 .$$

where Y, X, Y_1, Y_2, n are matrices corresponding to the vectors y_t, x_t, y_{t+1}, y_{t+2} and y_{t+3}, respectively. Notice that for $M_i < G$ the corresponding parameter matrix Δ_i will have $G - M_i$ zero columns. The assumptions on the disturbance term are,

$$E(u_t) = 0$$

$$E(u_tu'_{t'}) = \begin{cases} 0, & t \neq t' \\ \Sigma, & t = t' \end{cases}$$

with Σ a positive definite contemporaneous covariance matrix. Finally, X and Y_i^* ($i = 1, 2, \ldots, n$) are assumed predetermined so that,

$$\text{plim } (\frac{X'u}{T}) = \text{plim } (\frac{Y_i^{*\prime}u}{T}) = 0 .$$

The reduced form for the system represented in equation (1) is

$$(2) \qquad y_t = x_t\Pi_0 + y^*_{t+1}\Pi_1 + \ldots + y^*_{t+n}\Pi_n + v_t$$

where as per the usual convention, $\Pi_0 = -\Gamma B^{-1}$, $\Pi_i = -\Delta_i B^{-1}$ ($i = 1, 2, \ldots, n$) and $v_t = -u_t B^{-1}$. On the basis of the assumptions for u_t, it is easily shown that

$$E(v_t) = 0$$

$$E(v_t v'_{t'}) = \begin{cases} 0, & t \neq t' \\ \Omega = (B^{-1})'\Sigma B^{-1}, & t = t' . \end{cases}$$

Imposing the rational expectations hypothesis, $y^*_t = E(y_t)$ it follows from equation (2) that

$$(3) \qquad y^*_t = \bar{x}_t\Pi_0 + y^*_{t+1}\Pi_1 + \ldots + y^*_{t+n}\Pi_n$$

where $\bar{x}_t = E(x_t)$.[4] On the basis of equation (3), expressions for the expectations variables which depend upon the predetermined variables in future periods and the reduced form parameters can be developed. That is, for simplicity assume $n = 3$ and from equation (3) then,

$$(4i) \qquad y^*_{t+1} = \bar{x}_{t+1}\Pi_0 + y^*_{t+2}\Pi_1 + y^*_{t+3}\Pi_2$$

$$(4ii) \qquad y^*_{t+2} = \bar{x}_{t+2}\Pi_0 + y^*_{t+3}\Pi_1$$

and

(4iii) $y^*_{t+3} = \bar{x}_{t+3}\Pi_0$

where $\bar{x}_{t+i} = E(x_{t+i}/t)$ is the expected value of x_{t+i} conditioned on information available at time period t. The observations on the conditioned projections, $\bar{x}_{t+i}$, are written as the matrix $\bar{X}_i$. It is assumed that the projected exogenous variables are predetermined in the system, i.e., that plim $(\frac{\bar{X}_i' u}{T}) = 0$. Also, and importantly observe that for writing y^*_{t+1}, y^*_{t+2}, y^*_{t+3} it has been assumed values outside the planning period, n = 3, can be omitted.

Using equations (4i), (4ii), (4iii) to substitute for the unobservable variables in equation (3) and collecting terms on the current and future expected values for the predetermined variables yields,

(5) $$y^*_t = \bar{x}_t\Pi_0 + \bar{x}_{t+1}\Pi_0\Pi_1 + \bar{x}_{t+2}(\Pi_0\Pi_1^2 + \Pi_0\Pi_2)$$
$$+ \bar{x}_{t+3}(\Pi_0\Pi_1^3 + \Pi_0\Pi_2\Pi_1 + \Pi_0\Pi_1\Pi_2 + \Pi_0\Pi_3)$$

or

(5') $$y^*_t = \bar{x}_t\Pi_0 + \bar{x}_{t+1}\Pi_0(\Pi_1) + \bar{x}_{t+2}\Pi_0(\Pi_1^2 + \Pi_2)$$
$$+ \bar{x}_{t+3}\Pi_0(\Pi_1^3 + \Pi_2\Pi_1 + \Pi_1\Pi_2 + \Pi_3) .$$

Equation (5') puts a structure on the expectations variables y^*_{t+i} that can be used in the estimation of the model. This is shown by making the substitution of equation (5) into the reduced form, equation (2), and again deleting terms involving explanatory variables with subscripts greater than t + 3. The result is,

$$(6) \quad y_t = x_t\Pi_0 + \bar{x}_{t+1}\Pi_0\Pi_1 + \bar{x}_{t+2}\Pi_0(\Pi_1^2 + \Pi_2)$$

$$+ \bar{x}_{t+3}\Pi_0(\Pi_1^3 + \Pi_2\Pi_1 + \Pi_1\Pi_2 + \Pi_3) + v_t$$

Not surprisingly, the expected value for equation (6) is equation (5'), the equation which generates the unobservable value for y_t^*. Equation (6) also shows that y_t can be in principle estimated as a linear function (albeit not in the unrestricted reduced form parameters) of the current and conditioned future systematic components of the predetermined variables.

Possibilities for structures generating x_t and thus the conditioned systematic components of the predetermined variables, x_{t+i}, range from time series analysis to judgmental forecasts (Granger and Newbold [1977]).[5] For the present, it will suffice to simply specify the properties of the process. Assume that the process for the x_t is covariance stationary with $E[E(x_{t+i}/t)] = E(x_{t+i})$ a constant function in time. Also, recall that the x_t are predetermined so that the process is independent of the one generating v_t. Two interesting special cases exist. First, suppose that the information at time t is independent of the realization of the process at t+i, then $\bar{x}_{t+i} = E(x_{t+i}/t) = E(x_{t+i})$. For a first order autoregressive process in the x_t this would be equivalent to assuming the autoregressive parameters, say ρ_k (k = 1, 2, ..., K), equal zero. In this case, the arguments involving $\bar{x}_{t+i}$ in equation (6) can be omitted since by virtue of expressing the explanatory variables in deviation form, $E(x_{t+i}) = 0$ given that x_{t+i} is generated by a stationary process. In the second case there is a conditioned systematic effect with information at t influencing x_{t+i} so that in general $E(x_{t+i}/t) \neq E(x_{t+i})$.[6] In applied contexts, the lack of reliable basis for predicting the predetermined variables may lead to the use of something less than "complete" rational expectations.

Three special cases of interest in this regard are complete rationality, partially informative rationality and uninformative rationality in expectations generation. For illustrating these three cases assume a partitioning on $\bar{x}_{t+i} = (\bar{x}_{1t+i}, \bar{x}_{2t+i})$ where $\bar{x}_{1t+i}$ is of dimension K_1 and $\bar{x}_{2t+i}$ of dimension $K - K_1$. Let $\bar{x}_{1t+i}$ represent those predetermined variables with the systematic components that can be projected based on the information available at time t. Relatedly, let $\bar{x}_{2t+i}$ represent the explanatory variables for which information at t is independent of the realization at time t+i. Complete rationality then corresponds to the case in which $K_1 = K$ and the conditioned systematic components of all of the explanatory variables can be projected, e.g., equation (6).

Partially informative rationality refers to the situation in which $0 < K_1 < K$ and processes for projecting some but not all of the explanatory variables exist. This situation is easily incorporated into equation (6) by correspondingly partitioning Π_0' [= $(\Pi_{0_{11}}', \Pi_{0_{21}}'$]. Equation (6) then becomes,

$$(7) \qquad y_t = x_t\Pi_0 + (\bar{x}_{1t+1}, \bar{x}_{2t+1})\begin{pmatrix}\Pi_{0_{11}} \\ \Pi_{0_{21}}\end{pmatrix}\Pi_1$$

$$+ (\bar{x}_{1t+2}, \bar{x}_{2t+2})\begin{pmatrix}\Pi_{0_{12}} \\ \Pi_{0_{22}}\end{pmatrix}(\Pi_1^2 + \Pi_2)$$

$$+ (\bar{x}_{1t+3}, \bar{x}_{2t+3})\begin{pmatrix}\Pi_{0_{13}} \\ \Pi_{0_{23}}\end{pmatrix}(\Pi_1^3 + \Pi_2\Pi_1 + \Pi_1\Pi_2 + \Pi_3) + v_t .$$

Alternatively, the model can be written

$$(8) \qquad y_t = x_t\Pi_0 + \bar{x}_{1t+1}\Pi_{0_{11}}\Pi_1 + \bar{x}_{1t+2}\Pi_{0_{12}}(\Pi_1^2 + \Pi_2)$$

$$+ x_{1t+3}\Pi_{0_{13}}(\Pi_1^3 + \Pi_2\Pi_1 + \Pi_1\Pi_2 + \Pi_3) + v_t$$

where

$$0 = \bar{x}_{2t+1}\Pi_{0_{21}}\Pi_1 + \bar{x}_{2_{t+2}}\Pi_{0_{22}}(\Pi_1^2 + \Pi_2)$$

$$+ \bar{x}_{2t+3}\Pi_{0_{23}}(\Pi_1^3 + \Pi_2\Pi_1 + \Pi_1\Pi_2 + \Pi_3)$$

Since $\bar{x}_{2t+i}$, i = 1, 2, 3 is equal to zero. This model is rational but not completely informative because processes permitting projections for $\bar{x}_{2t+i}$ at values different than the mean (which equals zero) do not exist. It should not be confused with models which adopt <u>ad hoc</u> structures for the unobservable variables (McCallum [1976b]) or ones in which the decision on the process for generating y_t^* has been based on other criteria.

The noninformative rational expectations case is the one for which $K_1 = 0$ and the conditioned systematic components for x_t are projected as $E(x_t) = 0$. The appropriate expression is then

$$(9) \qquad y_t = x_t\Pi_0 + v_t \ .$$

In this case the rational expectation would be simply the current conditioned value for y. No forward looking process would be involved in the generation of the expectations variables. It is not difficult to see that this could be a rational approach when attempts to obtain projected values of the conditioning variables prove completely unreliable.

For applied work then the choice between models (6), (8) and (9) should be based on the relevant planning period for the system and the degree of confidence with which the decision makers involved can predict the predetermined variables. The recent work by Popkin [1975] and Feldstein [1971] would indicate that the incomplete model, recognizing the fact that explanatory variables for many economic models are difficult to project, may be the more realistic alternative.

3. Estimation

Two approaches can be employed to obtain consistent estimates of the structural parameters for the model in equation (1). The first and most obvious is indicated by making the analogy with the unobservable variables estimation procedures by Zellner [1970] and Goldberger [1971]. The second involves instrumental variables methods. Two options will be illustrated in this context, the general efficient estimators (Brundy and Jorgenson [1971, 1974]) and iterated instrumental variables (Lyttkens [1974]).

Unobservable Variables

The analogy to the unobservable variables problem is made by considering the structural equation (1) and presuming that there exist proxies for the unobservable variables y^*_{t+1}, y^*_{t+2}, y^*_{t+3}. For illustration, let these proxies be z_{1t}, z_{2t} and z_{3t}, respectively. The notation is chosen to emphasize that the z_{it} must be observable at time t. Following Zellner and Goldberger, the relationship between the unobservable and observed proxy variables is written as

$$(10) \qquad z_{it} = y^*_{t+i} + \xi_{it}$$

for i = 1, 2, 3. The rationality hypothesis is introduced through the use of equations (4i, ii and iii). Specifically, using these equations and making appropriate substitutions gives,

(11i) $y^*_{t+1} = \bar{x}_{t+1}\pi_0 + \bar{x}_{t+2}\pi_0\pi_1 + \bar{x}_{t+3}\pi_0(\pi_1^2 + \pi_2)$

(11ii) $y^*_{t+2} = \bar{x}_{t+2}\pi_0 + \bar{x}_{t+3}\pi_0\pi_1$

(11iii) $y^*_{t+3} = \bar{x}_{t+3}\pi_0$

as the structures for generating the unobservable variables.[7] For the covariance structure between ξ_t and u_t which is assumed by Goldberger [1972] and a process for generating projections of the $\bar{x}_{t+i}(i = 1, 2, 3)$ that produces errors independent of u_t and ξ_t, maximum likelihood estimation of the structural parameters for the model as specified in equation (1) can be obtained. The problem with the analogy to the Goldberger-Zellner unobservable variables model is with the use of the proxy. It would appear that consistent estimators for the structural parameters are available without the superfluous step involving the proxies.

Instrumental Variable Method

In two comparatively recent papers, Brundy and Jorgenson [1971, 1974] have shown that a class of efficient instrumental variables for use in estimating simultaneous equations can be delineated. The major result holds that an efficient set of instruments for a particular equation in the system must contain the included predetermined variables as instruments for themselves and instruments for the included endogenous variables computed using a consistent estimator for the appropriate partition of the reduced form parameter matrix (Brundy and Jorgenson [1971, pp. 209-211]).

One possibility for obtaining the instruments is equation (6). Again, assume that x_t and the projected $\bar{x}_{t+i}$ are predetermined. Equation (6) is non-linear in the reduced form parameters, and could be estimated with appropriate non-linear procedures, producing consistent estimates of the

reduced form parameters. However, the non-linear estimation is not attractive because of the computational problems given present software. An alternative would be to reparameterize equation (6) with $\alpha_0 = \Pi_0$, $\alpha_1 = \Pi_0\Pi_1$, $\alpha_2 = \Pi_0(\Pi_1^2 + \Pi_2)$ and $\alpha_3 = \Pi_0(\Pi_1^3 + \Pi_2\Pi_1 + \Pi_1\Pi_2 + \Pi_3)$, making the model linear in the parameters. The reduced form equation (6) can then be estimated by OLS to produce consistent $\hat{\alpha}_0$, $\hat{\alpha}_1$, $\hat{\alpha}_2$, $\hat{\alpha}_3$. Substituting these estimators into equations (11i), (11ii), (11iii) gives

(12i) $\quad \hat{y}^*_{t+1} = \bar{x}_{t+1}\hat{\alpha}_0 + \bar{x}_{t+2}\hat{\alpha}_1 + \bar{x}_{t+3}\hat{\alpha}_2$

(12ii) $\quad \hat{y}^*_{t+2} = \bar{x}_{t+2}\hat{\alpha}_0 + \bar{x}_{t+3}\hat{\alpha}_1$

and

(12iii) $\quad \hat{y}^*_{t+3} = \bar{x}_{t+3}\hat{\alpha}_0$.

as estimates for the unobservable variables y^*_{t+i}.

Also using the same reparameterization, fitted values for y_t can be directly computed from (6)

(13) $\quad \hat{y}_t = x_t\hat{\alpha}_0 + \bar{x}_{t+1}\hat{\alpha}_1 + \bar{x}_{t+2}\hat{\alpha}_2 + \bar{x}_{t+3}\hat{\alpha}_3$.

Thus, with appropriate assumptions for the process generating x_t and related estimates for $\bar{x}_{t+i}$, both $\hat{y}^*_{t+i}$ from (12) and $\hat{y}_t$ from (13) can be used as instruments for estimating the structural parameters of equation (1).

For example, the first structural equation from system (1) would be written

(14) $\quad y_{1t} = y_{2t}\beta_{.1} + x_{1t}\Gamma_{.1} + y^*_{t+i}\Delta_{i.1} + u_t$

where the exclusion restrictions have been reflected in the vectors $\beta_{.1}$, $\Gamma_{.1}$ and $\Delta_{i.1}$ of the matrices β, Γ and Δ_i and only one set of expectations

variables, for period t+i, has been included. The instrumental variable estimator is

$$(15)\quad \begin{bmatrix} \hat{\beta}_{.1} \\ \hat{\Gamma}_{.1} \\ \hat{\Delta}_{i.1} \end{bmatrix} = [(\hat{y}_{2t}, x_{1t}, \hat{y}^*_{t+i})'(y_{2t}, x_{1t}, y^*_{t+i})]^{-1}[(\hat{y}_{2t}, x_{1t}, \hat{y}^*_{t+i})'y_{1t}]$$

$$= \begin{bmatrix} \hat{y}_{2t}'y_{2t} & \hat{y}_{2t}'x_{1t} & \hat{y}_{2t}'y^*_{t+i} \\ x_{1t}'y_{2t} & x_{1t}'x_{1t} & x_{1t}'y^*_{t+i} \\ \hat{y}^*_{t+i}'y_{2t} & \hat{y}^*_{t+i}'x_{1t} & \hat{y}^*_{t+i}'y^*_{t+i} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \hat{y}_{2t}'y_{1t} \\ x_{1t}'y_{1t} \\ \hat{y}^*_{t+i}y_{1t} \end{bmatrix}$$

$$= \begin{bmatrix} \hat{y}_{2t}'\hat{y}_{2t} & \hat{y}_{2t}'x_{1t} & \hat{y}_{2t}'\hat{y}^*_{t+i} \\ x_{1t}'\hat{y}_{2t} & x_{1t}'x_{1t} & x_{1t}'\hat{y}^*_{t+i} \\ \hat{y}^*_{t+1}'\hat{y}_{2t} & \hat{y}^*_{t+i}'x_{1t} & \hat{y}^*_{t+i}'\hat{y}^*_{t+i} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \hat{y}_{2t}'y_{1t} \\ x_{1t}'y_{1t} \\ \hat{y}^*_{t+i}y_{1t} \end{bmatrix}$$

if,

$$(16i)\quad \hat{y}_{2t}'y_{2t} = \hat{y}_{2t}'\hat{y}_{2t}$$

$$(16ii)\quad \hat{y}_{2t}'y^*_{t+i} = \hat{y}_{2t}'\hat{y}^*_{t+i}$$

$$(16iii)\quad x_{1t}'y_{2t} = x_{1t}'\hat{y}_{2t}$$

$$(16iv)\quad x_{1t}'y^*_{t+i} = x_{1t}'\hat{y}^*_{t+i}$$

$$(16v)\quad \hat{y}^*_{t+i}y_{2t} = \hat{y}^*_{t+i}\hat{y}_{2t}$$

$$(16vi)\quad \hat{y}^*_{t+i}y^*_{t+i} = \hat{y}^*_{t+i}\hat{y}^*_{t+i}\,.$$

Equations (16i), (16iii) and (16v) follow by properties of OLS residuals. However, (16ii), (16iv) and (16vi) are true only if $[X', x_{t+i}']\hat{\xi}_i = 0$ where

$\hat{\xi}_i$ is defined as $\hat{\xi}_i = y^*_{t+i} - \hat{y}^*_{t+i}$. Conditions (16) imply that (15) can be written

$$(17)\qquad \begin{bmatrix} \hat{\beta}_{.1} \\ \hat{\Gamma}_{.1} \\ \hat{\Delta}_{i.1} \end{bmatrix} = [(\hat{y}_{2t}, x_{1t}, \hat{y}^*_{t+i})'(\hat{y}_{2t}, x_{1t}, \hat{y}^*_{t+i})]^{-1} \times [(\hat{y}_{2t}, x_{1t}, \hat{y}^*_{t+i})'y_{1t}].$$

Equation (17) is the OLS estimator of the first structural equation where y^*_{t+i} and y_{2t} have been replaced by their estimates $\hat{y}^*_{t+i}$ and $\hat{y}_{2t}$. Therefore the OLS estimator (17) is equivalent to the IV estimator (15) only if the conditions stated in (16) hold. When these conditions hold then the results of Brundy and Jorgenson [1971-1974] apply and IV/OLS estimation of the structure yields consistent and efficient structural estimators. However, the conditions (16) imply that either $\hat{y}^*_{t+i}$ is equal to y^*_{t+i} with probability one, or that the error generated, $\hat{\xi}_i$, is orthogonal to the predetermined variables of the system. These conditions may be rather strong given that the exclusion restrictions have not been incorporated in the estimation of the parameters used to compute $\hat{y}^*_{t+i}$. If they do not hold, then the estimates may not be efficient or even consistent.

Iterated Instrumental Variables

The iterated instrumental variables procedure is a special type of estimator which is efficient and perhaps performs better than IV estimation for small samples (Brundy and Jorgenson [1974], Lyttkens [1974]). To produce this estimator, initial values for y_{2t}, y^*_{t+i} in equation (14) are required. Although a number of possibilities for these initial values exist, natural candidates are ${}^0\hat{y}_{2t}$ and ${}^0\hat{y}_{t+i}$, computed from equations (12) and (13) (the superscripts will indicate the iteration number). With the initial values, equation (14) can be estimated provisionally using the estimator shown in

equation (17) applying this process for each of the structural equations will produce the structural estimates ${}^{1}\hat{\beta}$, ${}^{1}\hat{\Gamma}$ and ${}^{1}\hat{\Delta}_i$, which can be used to derive the reduced form parameter matrices ${}^{1}\hat{\Pi}_0 = - {}^{1}\hat{\beta}\,{}^{1}\hat{\Gamma}^{-1}$, ${}^{1}\hat{\Pi}_i = - {}^{1}\hat{\Delta}_i\,{}^{1}\hat{\Gamma}^{-1}$. These provisional reduced form parameter estimates along with the observed and projected exogenous variables can be used to compute second set of values for y_{2t} and y^{*}_{t+i}, $\hat{y}_{2t}$ and $\hat{y}^{*}_{t+i}$, using, respectively, equations (11i), (11ii), (11iii) and either equation (6) or (2). The process beginning with equation (14) is then started again, new estimates of the structural and reduced form parameters are obtained. The iterative process is continued until the estimates differ by arbitrarily small values between iterations.

The IIV method is preferred because upon convergence, it is reasonable to assume that $\hat{y}^{*}_{t+i} = y^{*}_{t+i}$ with probability one. Upon convergence, all of the prior restrictions have been incorporated in the estimation of the parameters used to compute $\hat{y}^{*}_{t+i}$, implying the use of a true and information rich instrumental variable. Lyttkens has shown that for normality of the structural disturbances the resulting estimates approximate full information maximum likelihood estimates. Again, the results follow if the projected values $\bar{x}_{t+1}$, $\bar{x}_{t+2}$, $\bar{x}_{t+3}$ can be viewed as predetermined. The iterated instrumental variables procedure is a special case of the above-mentioned class of efficient estimates because after the first stage, reduced estimators are used in computing the included right hand side endogenous variables.

Numerical results obtained by Brundy and Jorgenson [1974] indicate that the iterated instrumental variables estimators may be efficient relative to other instrumental variables methods for small samples. It also turned out that in the case these authors studied, there was little advantage in full systems methods and/or more than one iteration. Given that the object of rational expectations is to use y^{*}'s consistent with the structure and these

exploratory sampling results, it would appear that the limited information iterated instrumental variables method is particularly appropriate for the problem at hand.

4. Rational and Adaptive Expectations and Dynamics

The connection between rational expectations and adaptive expectations is comparatively easily made in the context of the model set out in Section 2.[8] For the case of non-informative rationality and G = 1, equation (9) shows that

$$(18) \qquad y_t^* = \bar{x}_t \pi_0$$

with π_0 in this situation a vector. If the predetermined variables are lagged endogenous variables then equation (18) can be written

$$(19) \qquad y_t^* = \delta_0 + \sum_{i=1}^{q} \delta_i y_{t-i}$$

or in terms of the corresponding reduced form equations

$$(20) \qquad y_t = \delta_0 + \sum_{i=1}^{q} \delta_i y_{t-i} + v_t$$

which is a distributed lag. Thus expectations are formed in an extrapolative fashion. Equations (18), (19) and (2) show the speciality of the rational expectations structure which must occur for extrapolative expectations to be rational.

If the model in equations (19) and (20) is reparameterized, for q approaching infinity, $\delta_0 = 0$ and $\delta_i = (1-\lambda)\lambda^i$ for i = 1, 2, ... Then an adaptive expectations model results. That is

$$(21) \qquad y_t^* = (1-\lambda) \sum_{i=0}^{\infty} \lambda^i y_{t-1-i}$$

the reduced form of the simple adaptive expectations model (Nerlove [1958, 1972]).

Dynamic properties of this adaptive expectations model and the more general rational model can be easily developed. Consider again equation (3). It can be rewritten

$$(22) \qquad [y^*_t, y^*_{t+1}, y^*_{t+2}] = [y^*_{t+1}, y^*_{t+2}, y^*_{t+3}] \begin{bmatrix} \Pi_1 & I & 0 \\ \Pi_2 & 0 & I \\ \Pi_3 & 0 & 0 \end{bmatrix} + \bar{x}_t[\Pi_0 \; 0 \; 0] .$$

Define,

$$W_t = [y^*_t, y^*_{t+1}, y^*_{t+2}]$$

$$C = \begin{bmatrix} \Pi_1 & I & 0 \\ \Pi_2 & 0 & I \\ \Pi_3 & 0 & 0 \end{bmatrix}$$

and

$$\gamma = [\Pi_0 \; 0 \; 0] .$$

Then equation (21) becomes,

$$(23) \qquad W_t = W_{t+1}C + \bar{x}_t\gamma$$

or alternatively,

$$(24) \qquad W_{t+1} = W_tC^{-1} - \bar{x}_t\gamma C^{-1}$$

if C is nonsingular. Assuming that there are no lagged variables in the vector $\bar{x}_t$, the dynamic properties of the rational expectations will depend upon C^{-1}. The rational expectations, in particular, will be stable only if all the characteristic roots of $[C^{-1}]'$ are in the unit circle (Chow [1975]).

5. An Example Using a Simple Market Model

Consider the price determination process in a market of a particular commodity. Because of production lags, the specification for the supply equation involves current price as well as an expected price one time period in advance. Alternatively, this means that the planning period corresponds to $n = 1$, and that all events occurring more than one period ahead of time are not relevant in the formulation of expectations. Also, expectations are assumed homogeneous in the market. The demand side is assumed to involve only current price. In general, a model for such a market can be specified as,

$$(25) \qquad S_t = \alpha_0 + P_t\alpha_1 + P^*_{t+1}\alpha_2 + SS_t\,\alpha_3 + w_t$$

$$(26) \qquad D_t = \beta_0 + P_t\beta_1 + DS_t\,\beta_2 + \eta_t$$

and

$$(27) \qquad S_t = D_t + NT_t$$

where S_t is the quantity supplied, P_t current price, P^*_{t+1} expected price one period ahead, SS_t is a supply shifter (e.g., weather, costs, etc.), D_t is the quantity demanded, DS_t a demand shifter (e.g., income, price of close substitutes, etc.), NT_t is net export, and w_t and η_t random disturbances. The model includes no storage so that production and the quantity supplied are the same. A similar interpretation follows for consumption and the quantity demanded.

In matrix form and corresponding to equation (1), the structure of the model can be written as

$$(29) \qquad [S_t\ D_t\ P_t]B + [1\ SS_t\ DS_t\ NT_t]\Gamma + [S^*_{t+1}\ D^*_{t+1}\ P^*_{t+1}]\Delta_1 + u_t = 0$$

where

$$B = \begin{bmatrix} -1 & 0 & -1 \\ 0 & -1 & 1 \\ \alpha_1 & \beta_1 & 0 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} \alpha_0 & \beta_0 & 0 \\ \alpha_3 & 0 & 0 \\ 0 & \beta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha_2 & 0 & 0 \end{bmatrix}$$

and

$$u_t = [n_t \; w_t \; 0].$$

The reduced form for the structure shown in equation (29) is,

$$(30) \qquad [S_t \; D_t \; P_t] = [1 \; SS_t \; DS_t \; NT_t]\Pi_0 + [S^*_{t+1} \; D^*_{t+1} \; P^*_{t+1}]\Pi_1 + v_t$$

where $\Pi_0 = -\Gamma B^{-1}$, $\Pi_1 = -\Delta_1 B^{-1}$ and $v_t = -u_t B^{-1}$.

Applying the rational expectations hypothesis, we have, as previously described,

$$(31) \qquad [S^*_{t+1} \; D^*_{t+1} \; P^*_{t+1}] = [1 \; \overline{SS_{t+1}} \; \overline{DS_{t+1}} \; \overline{NT_{t+1}}]\Pi_0$$

where terms with time subscripts greater than t+1 have been omitted. Substituting equation (31) into the reduced form produces the expression

$$(32) \qquad [S_t \; D_t \; P_t] = [1 \; SS_t \; DS_t \; NT_t]\Pi_0 + [1 \; \overline{SS_{t+1}} \; \overline{DS_{t+1}} \; \overline{NT_{t+1}}]\Pi_0\Pi_1 + v_t$$

Assuming as previously that all variables are measured as derivations from their means, the model can be alternatively expressed as

$$(29') \qquad [S_t \; D_t \; P_t]B + [SS_t \; DS_t \; NT_t]\Gamma + [S^*_{t+1} \; D^*_{t+1} \; P^*_{t+1}]\Delta_1 + u_t = 0$$

$$(30') \qquad [S_t \; D_t \; P_t] = [SS_t \; DS_t \; NT_t]\Pi_0 + [S^*_{t+1} \; D^*_{t+1} \; P^*_{t+1}]\Pi_1 + v_t$$

$$(31') \qquad [S^*_{t+1} \; D^*_{t+1} \; P^*_{t+1}] = [\overline{SS_{t+1}} \; \overline{DS_{t+1}} \; \overline{NT_{t+1}}]\Pi_0$$

and

$$(32') \quad [S_t \; D_t \; P_t] = [SS_t \; DS_t \; NT_t]\Pi_0 + [\overline{SS_{t+1}} \; \overline{DS_{t+1}} \; \overline{NT_{t+1}}]\Pi_0\Pi_1 + v_t \, .$$

The connection between models with and without constant terms is established. The latter is more convenient since, as is clear in equation (32), the constant appears twice in the expression making a collection of terms necessary.

Depending on whether or not $\overline{SS_{t+1}}$, $\overline{DS_{t+1}}$ and $\overline{NT_{t+1}}$ can be forecasted with a reasonable degree of confidence, there results a fully informed, partially informed or uninformed rational expectation model. To illustrate this earlier classification system observe that at time t the realized values of SS_t, DS_t, NT_t are known with probability one. Alternatively, $\overline{SS_{t+1}}$, $\overline{DS_{t+1}}$ and $\overline{NT_{t+1}}$ provided that their expected values conditioned on the basis of information at t are not zero, must be estimated. These values are estimated on the basis of the processes assumed for SS_t, DS_t and NT_t. Suppose for simplicity that $E(NT_{t+1}/t) = E(NT_t) = 0$ and $E(SS_{t+1}/t) = E(SS_t) = 0$ but that $DS_t = \rho DS_{t-1} + \varepsilon_t$ so that $E(DS_{t+1}/t) = \rho DS_t$. With ε_t normally and independently distributed with mean zero and variance σ^2 and $|\rho|<1$, the estimate for $\overline{DS_{t+1}}$ would be $\hat{\rho}DS_t$ with $\hat{\rho}$ perhaps an OLS estimator for ρ. Thus, the ability to predict the exogenous variables, the quality of the estimate for $\hat{\rho}$ in this case, has a direct influence on the specification of models describing systems in which expectations about the future are important. In this regard, the rational expectation approach represents an interesting way of investigating the influence of information--in the form of projected values of explanatory variables--on the behavior of systems.

6. Conclusions

These results extend those of Nelson [1975b] and McCallum [1976b] to show that rational expectations can be used in econometric models while retaining consistent estimators of the structural parameters. The estimators which result from the application of the recommended iterative instrumental variables approach are consistent (like those of McCallum) but not partially rational. The McCallum estimators are in fact like those which would result from the unobservable variables model discussed in Section 3 but with an ad hoc structure for the y^*_{t+j}. The present method is more direct and consistent with the rational expectations hypothesis. We would conjecture that the estimators obtained are more efficient but substance for this point will have to await more analytical or capital intensive work.

Apart from the question of estimation, the development has shown how rational expectations relates to other approaches to the problem of generating unobservable variables. The concepts of complete, partially informed and noninformative rationality should help to reconcile the various approaches to expectations formulation by showing what they imply in terms of the rational approach. It would appear that many of the pragmatic expectations generating mechanisms are rational provided that the predetermined variables cannot be predicted and that the endogenous variables considered are generated by an autoregressive process.

Finally, the results point to two interesting problems in applying the model. The first is multicollinearity in the reduced form due to the lack of variation among the projected predetermined variables. The observation to be made in this connection is that if the projected variables are colinear then not much is being gained from the present model over one with

a shorter planning period. In short, severe multicollinearity problems would suggest that a model with a shorter time frame be adopted. The second problem relates to the ability to project the exogenous variables with accuracy. If this is not possible then a model with a shorter time frame may be appropriate. Both difficulties have interesting implications in that they provide some empirical guidelines for naturally limiting the time frame and complexity of econometric models of systems in which expectations play an integral role.

FOOTNOTES

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[1]The rational expectations hypothesis implies that each unobservable expectations variable must have a counterpart endogenous variable (although not necessarily contemporaneously) in the system or model.

[2]The predetermined variables are assumed not to include lagged endogenous variables. The problem for study is thus like the one examined by Nelson [1975b] and McCallum [1976b]. Inclusion of lagged endogenous as predetermined variables adds substantially to the complexity of the problem by providing for what is essentially an adaptive structure.

[3]Events occurring beyond t+n time periods will not be included in the model. This assumption is crucial to the results to follow.

[4]Of course, if the x_t are exogenous then $E(x_t) = x_t$.

[5]Methods for time series analysis and composite forecasting are nicely outlined in Granger and Newbold [1977] and outside the scope of the present discussion. The point to be made is that some process for generating the future values for the explanatory variables must be available. As will be subsequently apparent, the unavailability of such a process would truncate the expectations mechanism.

[6]The effect of including estimates of $\bar{x}_{t+i}$ in equation (6) and related errors in variables problems is deferred to the subsequent section.

[7]Here and for the IV estimators, it should be clear that multicollinearity may present problems. The $\bar{x}_{t+i}$ are likely to be highly interrelated. Also the methods for projecting or estimating the $\bar{x}_{t+i}$ may by their similarity compound the problem. What this suggests is that models with longer planning periods are likely to present estimation difficulties.

[8]A similar formulation can be developed by assuming that the x_t are generated by an extrapolative process. This is easily seen by considering equation (18) and assuming $x_t = r \sum_{i=1}^{\infty} \lambda^i x_{t-i} + n_t$ with appropriate conditions on n_t the random disturbance and the parameter λ.

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